

Name: Solutions

1. Find $\frac{dy}{dx}$ given $y = \sin(3x+1)$

$$\frac{dy}{dx} = \cos(3x+1) \cdot 3 = 3\cos(3x+1)$$

2. Find $f'(x)$ given $f(x) = \cos(2x^3)$

$$f'(x) = -\sin(2x^3) \cdot 6x^2 = -6x^2 \sin(2x^3)$$

3. Find y' given $y = \tan(\sqrt{x})$

$$y' = \sec^2(\sqrt{x}) \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{\sec^2(\sqrt{x})}{2\sqrt{x}}$$

4. Find $\frac{dy}{dx}$ given $y = \sqrt{6x}$

$$\frac{dy}{dx} = \frac{1}{2}(6x)^{-\frac{1}{2}} \cdot 6 = 3(6x)^{-\frac{1}{2}} = \frac{3}{\sqrt{6x}}$$

5. Find $g'(x)$ given $g(x) = (5 - \cos(3x))^2$

$$g'(x) = 2(5 - \cos(3x))(-\sin(3x) \cdot 3) = -6\sin(3x)(5 - \cos(3x))$$

6. Find y' given $y = (2x-1)^3(3-4x)^5$

$$\begin{aligned} y' &= 3(2x-1)^2 \cdot 2(3-4x)^5 + (2x-1)^3 \cdot 5(3-4x)^4(-4) \\ &= 6(2x-1)^2(3-4x)^5 - 20(2x-1)^3(3-4x)^4 \end{aligned}$$

7. Find $\frac{dy}{dx}$ given $y = \csc\left(\frac{x}{3} - \pi\right)$

$$\begin{aligned}\frac{dy}{dx} &= -\csc\left(\frac{x}{3} - \pi\right) \cot\left(\frac{x}{3} - \pi\right) \cdot \left(\frac{1}{3}\right) \\ &= -\frac{1}{3} \csc\left(\frac{x}{3} - \pi\right) \cot\left(\frac{x}{3} - \pi\right)\end{aligned}$$

8. Find $k'(x)$ ^{error} given $k(x) = x \sec(3 - x^4)$

$$\begin{aligned}K'(x) &= \sec(3 - x^4) + x \cdot \sec(3 - x^4) \tan(3 - x^4) (-4x^3) \\ &= \sec(3 - x^4) - 4x^4 \sec(3 - x^4) \tan(3 - x^4)\end{aligned}$$

9. Find y' given $y = \frac{\sin(x^5)}{\cos(-x)}$

$$y' = \frac{\cos(-x) \cos(x^5) \cdot 5x^4 - \sin(x^5) (-\sin(-x) (-1))}{(\cos(-x))^2}$$

10. Find $\frac{dy}{dx}$ given ~~$y = \csc\left(\frac{x}{3} - \pi\right)$~~ $y = \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)^2$

$$\frac{dy}{dx} = 2\left(\frac{1}{\sqrt{x}} + \sqrt{x}\right) \left(-\frac{1}{2}x^{-3/2} + \frac{1}{2}x^{1/2}\right)$$

11. Find $p'(x)$ given $p(x) = \sqrt{\tan(2x)}$

$$p'(x) = \frac{1}{2} (\tan(2x))^{-1/2} \sec^2(2x) (2) = \frac{\sec^2(2x)}{\sqrt{\tan(2x)}}$$

12. Find y' given $y = \left(x^2 \sin\left(\frac{2}{x}\right)\right)^3$

$$y' = 3\left(x^2 \sin\left(\frac{2}{x}\right)\right)^2 \left(2x \sin\left(\frac{2}{x}\right) + x^2 \cdot \cos\left(\frac{2}{x}\right) \cdot (-2x^{-2})\right)$$

$$y - \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}}(x - \frac{1}{2}) \quad y - \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{3}(x - \frac{1}{2})$$

13. Multiple Choice: Find the equation of the line tangent to the curve at the point defined by $t = \frac{\pi}{6}$. The curve is defined parametrically by $x = \sin t$ and $y = \cos t$

a. $y - \frac{1}{2} = \frac{-\sqrt{3}}{3}\left(x - \frac{\sqrt{3}}{2}\right)$

b. $y - \frac{1}{2} = -\sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$

c. $y - \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{3}\left(x - \frac{1}{2}\right)$

d. $y - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x - \frac{1}{2}\right)$

e. $y + \frac{\sqrt{3}}{2} = \frac{-\sqrt{3}}{3}\left(x - \frac{1}{2}\right)$

$$x = \sin \frac{\pi}{6} = \frac{1}{2} \quad y = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\frac{dx}{dt} = \cos t \quad \frac{dx}{dt} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \frac{dy}{dt} = -\sin t \quad \frac{dy}{dt} = -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$y - \frac{\sqrt{3}}{2} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\left(x - \frac{1}{2}\right)$$

14. Find the equation of the line tangent to the curve at the point defined by $t = \frac{\pi}{3}$. The curve is defined parametrically by $x = \sec t$ and $y = \tan t$

$$x = \sec \frac{\pi}{3} = 2 \quad y = \tan \frac{\pi}{3} = \sqrt{3} \quad \frac{dx}{dt} = \sec t \tan t$$

$$\frac{dx}{dt} = \sec \frac{\pi}{3} \tan \frac{\pi}{3} = 2\sqrt{3} \quad \frac{dy}{dt} = \sec^2 t \quad \frac{dy}{dt} = \sec^2 \frac{\pi}{3} = 4$$

$$\frac{dy}{dx} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}} \quad y - \sqrt{3} = \frac{2}{\sqrt{3}}(x - 2)$$

Find $\frac{d^2y}{dx^2}$

15. Find $\frac{dy}{dx}$ given $y = \cos(5-x)$

$$\frac{dy}{dx} = -\sin(5-x)(-1) = \sin(5-x)$$

$$\frac{d^2y}{dx^2} = \cos(5-x)(-1) = -\cos(5-x)$$

Find $\frac{d^2y}{dx^2}$

16. Find $\frac{dy}{dx}$ given $y = \cos^2(5-x)$

$$\frac{dy}{dx} = 2\cos(5-x)(\sin(5-x)(-1)) = -2\cos(5-x)(\sin(5-x))$$

$$= -\sin(10-2x)$$

$$\frac{d^2y}{dx^2} = -\cos(10-2x)(-2) = 2\cos(10-2x)$$

17. Find y'' given $y = \left(\frac{1}{2}x^2 + 5\right)^3$

$$y' = 3\left(\frac{1}{2}x^2 + 5\right)^2(x) = 3x\left(\frac{1}{2}x^2 + 5\right)^2$$

$$y'' = 3\left(\frac{1}{2}x^2 + 5\right) + 3x(2x) = \frac{3}{2}x^2 + 15 + 6x^2 = \frac{9}{2}x^2 + 15$$